Modeling Vehicle Interactions during Freeway Ramp Merging in Congested Weaving Section

Xia Wan, Ph.D., (Corresponding Author)
Department of Civil and Environmental Engineering,
University of Wisconsin-Madison, Madison,
1415 Engineering Drive, USA, WI 53706,
Phone: 1-608-556-4289
E-mail: wan5@wisc.edu

Peter J. Jin, Ph.D., Postdoctoral Fellow,
Department of Civil, Architectural, and Environmental Engineering,
The University of Texas at Austin, Austin, TX 78701
Phone: 1-512-232-3124
Email: jjin@austin.utexas.edu

Fan Yang, Ph.D. Candidate,
Department of Civil and Environmental Engineering,
University of Wisconsin-Madison, Madison,
WI 53706, USA
E-mail: fyang29@wisc.edu

Bin Ran, Ph.D.
Professor
School of Transportation, Southeast University
No.2 Si Pai Lou, Nanjing 210096, China
and
Civil and Environmental Engineering
University of Wisconsin - Madison
Madison, WI 53706, USA
bran@wisc.edu

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ABSTRACT

The difficulty to capture the interactions between vehicles in congested merging area during lane change process hinders the development of microscopic traffic modeling. The main goal of this paper is to quantify the interactions between vehicles during lane change in congested weaving section and try to incorporate them into vehicles’ (merging vehicles, their putative leaders (PL) and putative followers (PF)) acceleration-deceleration models. Based on the findings by analyzing the US101 data, the yielding behavior of merging vehicles’ putative leader (PL) for merging cooperation and the lateral separation between vehicles in weaving section (both features largely ignored in the past research) are introduced into our proposed acceleration-deceleration models. The visual angle information is employed as the stimuli in our models to present the effect of lateral movement of vehicles. The particular car following behaviors are incorporated in the modeling, such as the PF gradually changing its car-following leader from PL to merging vehicle, which depends on the relative locations of vehicles.

The model calibration and validation results based on field data have demonstrated that the proposed acceleration-deceleration models qualitatively simulated the driving behavior of vehicles in the lane change process and obtained acceptable training and testing errors. To verify the cooperation behavior of PL in lane changing, a comparison result of the proposed PL acceleration-deceleration model and a base model implied that incorporating the effects of merging vehicle on PL into the models could enhance the realism of the lane changing model. Findings from this study could contribute to the understanding of interactions between vehicles during complex lane-changing behavior.

Key Words: Merging Process, Interactions between Vehicles, Yield Behavior, Lateral Separation, Congested Weaving Section
1. **INTRODUCTION**

The lane change behavior of vehicles is complex and the fundamental part of microscopic traffic flow simulation model, which has attracted increased attention recently. A significant number of works have been done to build microscopic merging behavior related models. However, the difficulty to capture the interactions between vehicles during a complex lane change behavior hinders the development of microscopic traffic simulation (1-4).

Only a limited number of studies in the literature deal with vehicle interactions in detail. In existing studies, to handle the vehicle interactions in lane change modeling, lane changes are usually classified to three types based on the gap distance between putative leader (PL) and putative follower (PF): Free Lane Change, Cooperative Lane Chang and Forced Lane Change (5-8). The key assumption of this method, the existence of interactions between vehicles mainly depends on the gap distance between PL and PF and the interactions only could trigger the PF to yield to merging vehicles, has some limitations. First, in our previous merging behavior analysis, in congested weaving section, the situation that the PL speed up to cooperate the merge process is frequently observed (9). On the other hand, interactions among the merging vehicle, its PL and its PF could not be unaffected by surrounding traffic environment. For example, the probability of yielding of PF is not only related with the locations of merging vehicle and PL, but also the vehicle closely following it, which could decrease the yielding possibility due to safety consideration. Thus, this paper aims to analyze and model the interactions between merging vehicles, their PL and PF, considering the effects of other vehicles around them.

In the real traffic world, every vehicle makes the decision and reacts based on its perceived intentions of the other vehicles. The lateral movement of vehicles in the lane change process was rarely analyzed in the past, which contains valuable information for other drivers’ perception and judgment. In the merge process, the merging vehicles have to follow their PL by accompanying with lateral movement while the PF need gradually to set the merging vehicle as its new following leader based on the lateral movement of itself. The lateral effects have been proved widely existing and influential in realistic car-following behavior (10, 11). Especially in the area- a weaving section we studied, there are three types of vehicles with different route plans: going through, merging in and exiting the highway mainline. Their different route plans lead to significant lateral separation between the follower and leader driving on the same lane. Angular velocity contains the lateral information, which was used by Kou to build acceleration-deceleration model of merging vehicles (12). This paper would adopt the visual angle information as the stimulus between vehicles during the merge process.

This study first analyzes driving behavior of vehicles in longitudinal and lateral directions with observed vehicle trajectory data, and then builds the dynamic acceleration-deceleration models for merging vehicle, PL and PF by incorporating two dimensional interactions. To simplify the complex lane change condition, the study only focuses on the interactions between vehicles after the merging vehicles merge in their accepted gap. The processes of the merging vehicles driving through their rejected gap are not included in this study, because these conditions are more like overtaking executions. Second, we present the literature review about interactions analysis in lane change and acceleration-deceleration modeling. The third section is the description of the NGSIM data and empirical driving behavior analysis in longitudinal and lateral directions. Fourth, it shows the acceleration-deceleration model of merging vehicle, PL and PF incorporating interactions between vehicles. The model calibration and testing are discussed in section five. The conclusion and future work are addressed in section six.
2. LITERATURE REVIEW

2.1 Interaction between Vehicles during Merging Action

In the merging area, there are two interactive traffic streams: merging vehicles and mainline driving-through vehicles. In the early lane change models, the complex dynamic interactive behavior is simplified by assuming that merging traffic has no influence on the mainline traffic (13, 14). Nevertheless, many observations indicate that driving-through vehicles express a kind of cooperative behavior by changing to the inner lanes or by yielding to create gaps for merging vehicles (15-17). It is clear that merging drivers adjust their speeds according to speeds of their putative leader (PL) and putative follower (PF) on the target lane. The presence of yield behavior of mainline vehicles indicates the existence of interaction between merging vehicles and mainline traffic during the lane change process. However, only a limited number of studies in literature deal with vehicle interactions in lane change modeling. To our best knowledge, though we noticed the PL shows yield behavior to merging vehicles in the congested merging area, most of the existing research only studies the yield behavior of the mainline lag vehicle (9).

Researchers applied the game theory to model the interaction between the merging vehicle and its PF during the merging process. Kita et al. (2002) modeled the interaction between the merging vehicle and its PF as a two-person non-zero-sum non-cooperative game with complete information (18). The merging vehicle decides whether to move-in or pass, and the mainline PF decides whether to give way or not. However, this research only considered the situation that the vehicle driving on the mainline changes to the inner lane to give way to merging vehicle, and it assumes the two vehicles having conflict keep constant speed during the merge process. Liu (2007) used the game theoretical approach to model merging and yielding behavior at freeway's on-ramp section (19). The strategy of the competition between merging vehicle and its PF is that the PF aims to maintain their initial car-following state and minimize speed variations, and the merging vehicle wants to join mainline traffic in the minimal time possible. A bi-level estimation methodology was used to search the Nash equilibrium for the two players. The demand on too much information in the game theoretic modeling limited its usage in the microscopic simulation.

Another way to modeling the interactions between vehicles is to divide the merge process as three types: free, cooperative and forced lane change. Hidas (2005) established a simulation model, called ARTEMiS, to model interactions by using the autonomous agent (7). In the model, when the gap is less than the given minimum free lane change gap, the PF may act as giving way, slowing down, or not giving way, whose willing depends on the level of congestion and the individual driver’s characteristics. The feasibility to slow down for the PF is calculated by the space gap between PF and the merging vehicle at the end of the deceleration period, but the effect of the vehicle behind the PF is not considered. The action choice (to yield or not to yield) of the PF is determined by the checking sequence of the model (when free lane change is impossible), since no vehicle communication is considered in the model. Ben-Akiva and Choudhury (2009) proposed a combined merging model which includes normal, courtesy and forced lane change (20). The model incorporated the courtesy deceleration of the PF if a normal lane change is impossible. The result of courtesy or forced lane change is modeled as instant deterministic choice of vehicles, ignoring the negotiations between drivers during the process.

Another method to realize interaction modeling is setting the interactions into the stimuli–response psychophysical concept and modifying the conventional car-following models to suit the lane change background. Sarvi (2007, 2011) built a freeway ramp merging micro-
simulation model, in which the acceleration-deceleration of merging vehicles and PF are linearly related with the stimuli from other vehicles (21, 22). For example, the acceleration-deceleration of PF is under the stimuli of the speed of merging vehicle, the speed of PL, distance between PF and merging vehicle and distance between PF and PL. The simulation models were calibrated with observed field lane change data.

Other researcher used distinct methods to model the interactions between vehicles. Wang (2005) presents an interaction-based model, in which the decision of PF (whether or not to provide courtesy yielding) is picked up randomly from a binominal distribution with a given probability parameter (8). Sun (2010) adopted a sequence of “hand-shaking” negotiations to handle the competition and cooperation among vehicles on arterial streets (1). Sun used the gap distance between merging vehicle and PF to determine that the PF yields or not to the merging vehicle.

In summary, most existing researches only model the possibility of the PF’s yielding behavior. This paper attempts to capture the interaction among the merging vehicles, PL and PF in merge process and model the courtesy yielding of PL. We choose the stimuli–response concept and car following theory to model the interactions because it can continuously apply the interactions between vehicles into the merging process modeling.

2.2 The Effect of Lateral Movement in the Acceleration-Deceleration Model

The longitudinal movement of vehicle in car following model is always a hot topic for researchers. The conventional car-following theory holds an assumption that vehicles travel in the middle of a single lane. However, the lateral separation of vehicle during the lane change process could not be ignored since the merging vehicle and its PL/PF drive on different lanes prior to the merge maneuver, which is more like the scenario under the staggered car following condition.

Recently, a few studies have been done focusing on the effect of lateral separation during car following. Gunay(2007) proved the existence of lateral discomfort during vehicles movement, and proposed the staggered car following theory, in which the car following movement of the following vehicle is under the impact of the off-center effects of its leader (10). Jin (2010) built a non-lane-based car following model to account the lateral separation characteristics between the leader and follower (11). The visual angle information was set as the stimulus during the car following condition.

Most lane change models assume the lane change execution is an instantaneous action after the gap selection, and they pay rare attention to the lateral separation of vehicles during interacting. However, the lateral movement of vehicle carries considerable information during the vehicle’s communication. Thus, in this report we attempt to introduce the effect of lateral separation into the lane change acceleration-deceleration modeling.

3. DATA SET AND EMPIRICAL ANALYSIS

3.1 General Description of NGSIM Data

This study uses vehicles trajectory data collected on a five-lane freeway section with an on-ramp from Ventura Boulevard and an off-ramp to Cahuenga Boulevard on U.S. Highway 101 (Hollywood Freeway), Log Angeles, California, USA (see figure 1a for the geometric layout). It is a part of FHWA’s Next Generation Simulation (NGSIM) program. The total length of the
observation area is 604 m, and the vehicle trajectories are updated in every 0.1 second from 7:50 to 8:35 a.m. on June 15, 2005. In this study, we focus on the weaving section, whose length is 212.25 m. In the 45 minutes observation time, the weaving section is in transition (7:50-8:05 am) and congestion (8:05-8:35 am) (23). The speeds of mainline traffic vary from 27.3 to about 51.50 km/h (average value is around 41.03 km/h) during the 45 minutes; while the average speed of the on-ramp merging vehicles when arriving the auxiliary lane is around 49.48 km/h. Thus, this data set could be identified as congested weaving section.

The NGSIM data set distinguishes three vehicle classes: motorcycles, cars, and trucks. Due to the low percentage of quantities of motorcycles and trucks in this data set (the total of them is less than 3 percent), they were excluded in this study. 398 merging vehicles were extracted out when a total of 11,779 vehicles were processed. In figure 1b, we sketched the study area and vehicle layout, where the merging vehicle (M) interacts with its putative leader (PL) and putative follower (PF) on the target lane and its leading vehicle (L) on the auxiliary lane. The leader of PL and the follower of PF also are investigated in this study, which are called PLL and PFF, respectively. In a weaving section, there are three types of vehicles having different route plans: driving-through vehicles, merging vehicles and exiting vehicles.

Merging vehicles’ merge gap selection and merge tactics are influenced by the traffic condition on the target lane. At the left of this section, we would analyze the effect of merging vehicle on mainline traffic under the condition of vehicle interactions.

![Data site-U.S.101](image)

![Vehicle layout and notations](image)

FIGURE 1 Data collection site (a) and the related vehicles (b).

3.2 Longitudinal Interactions during Merging Process

Previous lane change studies pointed out, when the gap between the PL and PF is smaller than minimum required gap for free lane change, the cooperative lane change condition is activated along with the yielding action of the PF. In this section, we explore the yielding behavior of
mainline traffic with field data from the congested area.

First, the proportion of two types yielding behavior of mainline traffic (slowing down or changing to the inner lane to create a gap for merging vehicle) is examined. Almost 3000 driving-through vehicles which once drive on the lane five (in figure 1a) are collected by screening based on their travel trajectory. The results show that there are only 51 of them changing to the inner lane in the 604 m observation area, and only 9 of these 51 could be considered as the yielding-based lane change vehicles, which were involved to be PFs or PLs in the study area. The possible reason for the low rate of yielding-based lane change in the observation area is that the anticipated yielding lane changes commonly occur prior to the effective ramp area, and the remaining driving-through vehicles either have to travel or intentionally persist in traveling on the merging vehicle target lane. Here, only the yielding behavior of mainline traffic vehicles by slowing down will be in detail analyzed and modeled, since the lane change-related yielding behavior rarely executes (manifested by its low percentage).

Merging vehicle getting into the gap between PL and PF would disturb the car-following status of them. Before the lane change maneuver, the PL needs to estimate the distance from the merging vehicle to itself to avoid collision for the anticipated merge, while the PF has to be ready for the anticipated merge action when it notices the strong merge intention of merging vehicle. During the merging, the merging vehicle switches to the PF’s new leader. We have noticed not only the PF is impacted by pressure of anticipated merge requirement, but also the PL shows cooperative yielding for merge vehicles. Fig. 2 shows two random examples of the merging process including yielding behavior of mainline traffic, during which the PL (PL of merging vehicle ID 2990) or PF (PF of merging vehicle ID 10864) obviously yields to merging vehicle, respectively. “0” at x axis in Figure 2 indicates the time point when merging vehicle gets into its accepted gap, and the end point of the time axis is the time point when the geometrical center point of the merging vehicle crosses the lane line shared by the auxiliary lane and target lane. The yielding behavior of the mainline traffic is manifested by dramatically acceleration (PL) or deceleration (PF) after the presence of merging vehicle, and the increase or decrease of the distance gaps between among them.

We use the change rate of gap distance between PLL and PL during merge process to quantify the yield behavior of PL. Excluding the PLL’s obvious deceleration case, in 67 merge processes, the gap distance between PLL and PL reduces by more than 15% percent due to the acceleration of PL for merge cooperation. Similarly, for the PF, in 86 merge processes the gap distance between PL and PF increases by more than 15% percent due to the deceleration of PF after removing the effect of the acceleration of PL. Considering that the total samples observed are 398, both of the yielding behavior of PL and PF should be included in the interaction modeling.
(a) Trajectory of interactive vehicles
(b) Trajectory of interactive vehicles
(c) Speed of interactive vehicles
(d) Speed of interactive vehicles
(e) Acceleration of interactive vehicles
(f) Acceleration of interactive vehicles

FIGURE 2 PL (merging vehicle ID 2990) and PF (merging vehicle ID 10864) Courtesy Yielding example.

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3.3 Lateral Interactions during Merging Process

In the merge process, when the PL and PF notice the existence of merging vehicle, they have the tendency to shift further away from the right side lane line to avoid the possible collision. Though the analysis results in section 3.2 show that only the PL of merging vehicle (ID 2990) is significantly affected by merging vehicle to perform yielding, figure 3a shows both the PL and PF shift further away from the right side lane line during the lane change process. The evidence indicates both the PL and PF are under the influence of the merging vehicle.

The U.S.101 data was further examined to reveal whether the lateral movement of PL and PF is a common phenomenon during merge process. We rescale the total time (between the time point when the merging vehicles start to involve in their accepted gap and the time point when they right merge into their target lane) and divide it to 20 sections of same length. Figure 3b and Figure 3c show the lateral movement of PL and PF for three different types

![Figure 3a: Lateral movement of vehicles during lane change (ID 2990)]

![Figure 3b: Average lateral movement of PL for different vehicles types](a)

![Figure 3c: Average lateral movement of PF for different vehicles types](b)

FIGURE 3 Lateral Interaction during the merging process.
of vehicles separately in such a weaving section. The figures show that the driving-through vehicles tend to keep away from the merging vehicle no matter what role they play (either as PL or PF), and their lateral movement are more noticeable right after they sense the presence of the merging vehicle. Meanwhile, the merged vehicle and exiting vehicle conduct lateral movement by coordinating with their route plan. Even the vehicles all travel on the target lane, there could be significant lateral separation between the leader and follower as the figure 3b and figure 3c show. After comparing the different lateral movement trajectories of PL and PF, we could tell that both the merged and exiting vehicles are closer to their own lateral destinations, respectively, at the same time point of the 20 point-scale dimension when they function as PL. This trend is reasonable since the PL travels longer than the PF across a merge event, which gives merged/exiting vehicles more time to move toward their target lateral destination.

The lateral movement analysis results illustrate that the effect of merging vehicle on its PL and PF exist and is manifested by the lateral movement of them during the lane change process. Another notable finding is that the lateral separation occurs frequently during the merge process, which should be included in the interaction modeling.

4.  MODELING

In this study, the interactions between vehicles are considered as continuous actions across the whole merge process. Because of the close relationship between lane change and car-following in the merging area, it is necessary to incorporate both of them in the merging process micro-simulation model. The merging vehicle getting onto the target lane leads to a distinct space reduction between its PL and PF and disturbance to current car-following condition. Before the merging vehicle arrives it accepted gap, its PL and PF maintain a car-following status under its own route plan. Immediately upon the emergence of the merging vehicle, the entire merging vehicle, PL and PF have to adjust current car-following condition and decide a set of moves to avoid potential collision for the following merge maneuver. The action strategies for vehicles during merge are assumed as follows:

- The merging vehicle (M): it sets its PL as its following vehicle for lane change preparation, and keeps a safe distance to its PF and its leading vehicle (L) on current lane;
- The PL: it follows its leading vehicle (PLL) and keeps a safe distance from the merging vehicle (M) for safety issue;
- The PF: it gradually changes its following vehicles from PL to M, and keeps a safe distance to the vehicle following it (PFF).

During a merge process in a weaving section, the merging vehicle, PL and PF do not travel on the middle of the same lane, thus the lateral separation between vehicles makes the conventional car-following model inapplicable in merge condition. Also, the PL and PF drivers also pay more attention to the merging vehicle’s lateral movement and use it to predict the movement of merging vehicle. New acceleration-deceleration models are built in this section based on visual angle information to simulate the acceleration-deceleration of the merging vehicle, PL and PF during the merging process, which considers the longitudinal space distance, lateral separation, speed difference and effect of different vehicles types.

4.1 Car Following with Visual Angle Information and Notations

During the lane change process, the main task of vehicles is to avoid collision with other vehicles and follow their route plan based on car-following. Many researchers reported that the dominant
perceptual factor (the stimulus) is the rate of change of visual angle in the car-following situation \((11, 12, 24)\). The change rate of visual angle, called angular velocity, could be expressed as \(\frac{d \theta(t)}{dt}\) in Figure 4.

When the leader and follower vehicles travel on the middle of the same lane (showed in Fig. 4a), the equation to calculate the visual angle \(\theta(t)\) and angular velocity are derived:

\[
\theta(t) \approx \frac{W_{n-1}}{X_{n-1}(t) - X_n(t)} \quad (1)
\]

\[
\frac{d \theta(t)}{dt} \approx \frac{W_{n-1}}{(X_{n-1}(t) - X_n(t))^2} \cdot [V_{n-1}(t) - V_n(t)] \quad (2)
\]

Where, \(W\) is the width of vehicle \(n-1\); \(X_{n-1}(t)\) and \(X_n(t)\) are the longitudinal location of vehicle \(n-1\) and vehicle \(n\), respectively, at time \(t\); \(V_{n-1}(t)\) and \(V_n(t)\) are the speed of vehicle \(n-1\) and vehicle \(n\), respectively, at time \(t\). The detailed derivation could be found in reference (19).

Considering the lateral separation during the lane change process, the visual angle and angular velocity shown in Figure 4b, with merging vehicle (M) and PL as an example, are modified as:

\[
\theta(t) \approx \frac{W_{PL}}{D(t)} \quad (3)
\]

\[
\frac{d \theta(t)}{dt} \approx \frac{W_{PL}}{D(t)^2} \cdot [V_{PL}(t) - V_M(t)] \quad (4)
\]

Where \(D(t)\) is the distance between PL and M at time \(t\), where \(D(t)\) is calculated as:

\[
D(t) = \frac{1}{2}(X_{PL}(t) - X_M(t))^2 + (Y_{PL}(t) - Y_M(t))^2 \quad (5)
\]

Where, \(Y_{PL}(t)\) and \(Y_M(t)\) are the lateral locations of PL and M, respectively, at time \(t\).

The fundamental psychophysical concept of car-following models is appropriate to model
acceleration-deceleration behavior with stimulus. The basic car following model is written as:

$$a_n(t + T) = \lambda \theta(t)$$

(6)

Where $a_n(t + T)$ is the acceleration of vehicle $n$ at time $t + T$; $T$ is the reaction time of driver.

4.2 Acceleration-Deceleration Modeling Incorporating Interactions between Vehicles

The driving behaviors of vehicles traveling in the weaving section are under the impact of the interactions between vehicles in the congested condition. The dynamic acceleration-deceleration of merging vehicle, PL and PF in the longitudinal direction are modeled in this section. The vehicles type information is contained in their lateral movement, serving as the input of these models.

4.2.1 Dynamic acceleration-deceleration model of merging vehicle

The acceleration and deceleration behavior of merging vehicles involves two tasks, including following the PL and keeping a safe distance from other vehicles around it to get appropriate space for lane change execution. Here, the PF and L (the leading vehicle on auxiliary lane) are set as the required safety space constraints for merging vehicle. The basic car-following model shown in equation 6 is linearly expanded to acceleration-deceleration model of merging vehicle, incorporating the car following (first term) and the influence of the PF (second term) and L (third term):

$$a_m(t + T) = \lambda_1 \theta_{PL}(t) - \lambda_2 \cdot \min[0, \theta_{DesPF} - \theta_{PF}(t)] + \lambda_3 \cdot \min[0, \theta_{DesL} - \theta_{L}(t)]$$

(7)

Where $\theta_{PL}$ is the visual angular velocity of PL from the merging vehicle’s view, calculated with equation 4; $\theta_{PF}(t)$ and $\theta_{L}(t)$ are the visual angle of PF and L, respectively, from the merging vehicle’s view at time $t$, calculated with equation 3; $\theta_{DesPF}$ and $\theta_{DesL}$ are the desired visual angle from merging vehicle to its PF and L, respectively, which are constant value needed to be calibrated in this model; $\lambda_1$, $\lambda_2$ and $\lambda_3$ are used to determine the weights of these three stimuli.

The first term of equation 7 represents the merging vehicle following its PL during the lane change process. The visual angular velocity $\theta_{PL}$ contains speed difference, longitudinal and lateral location information of M and its PL. The second term describes the response of merging vehicle to the close PF behind it, and its effect could only be acceleration. When the visual angle $\theta_{PF}(t)$ is less than the desired visual angle $\theta_{DesPF}$, it motivates the merging vehicle to speed up for the safety issue. The lateral separation information is contained in the visual angle. The third term represents the response of merging vehicle to its leading vehicle driving (L) on the auxiliary lane, which could only be deceleration stimuli for keeping safety space. The concept is the same as the second term, so here we skip the redundant description.

4.2.2 Dynamic acceleration-deceleration model of Putative Follower (PF)

During the merge process, the merging vehicle’s PF should change its following leader from PL to merging vehicle gradually. This process depends on the variation of relative lateral and longitudinal positions of vehicles.

In the lateral direction, when the merging vehicle is laterally close enough to PF, the PF would set merging vehicle as its new leader because of its strong merge intention. To capture this lateral distance effect, the lateral related position factor $\frac{\alpha l}{|V_M(t) - V_{PF}(t)|}$ is employed for the PF to determine its following leader. Here, $l$ is the average lane width of auxiliary lane and target lane,
and $\alpha$ is a parameter needed to estimate with field data. When $\frac{\alpha \cdot l}{|Y_M(t) - Y_{PF}(t)|}$ is large enough, the merging vehicle is the only leader of PL. Otherwise, the PF is under the stimulus coming from both the merging vehicle and PL.

In the longitudinal direction, if the merging vehicle is extremely close to the PF, it would results an unrealistically significant deceleration for PF in a conventional car-following model. Actually, the situation that the space between merging vehicle and its PF is extraordinarily tiny is common in the congested area when the merging vehicle meets a rejected gap prior to merging. To overcome this problem in car-following theory under lane change situation, a relative longitudinal location factor $\frac{\beta \cdot |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}$ is adopted here to present a resistance of PF to the unrealistic deceleration. This factor means, when plenty of space is left between merging and PL, instead of applying significant deceleration, the PF trusts the nearby merging vehicle would adjust its relative position prior to the lane change execution. $\beta$ is a parameter for calibration. On the other hand, the distance between PF and its follower (PFF) is incorporated in the model, which plays a crucial role for PF to check whether a yielding action is feasible or not.

Overall, the dynamic acceleration-deceleration model of PF, expressed by equation 8 and 9, is obtained.

$$
\begin{align*}
&\text{If } \frac{\alpha \cdot l}{\Delta Y_{M-PF}(t)} \geq 1, \\
&a_{PF}(t+T) = \lambda_4 \cdot \dot{\theta}_M(t) - \lambda_5 \cdot \min[0, \theta_{DesPFF} - \theta_{PFF}(t)] \\
&\text{If } \frac{\alpha \cdot l}{\Delta Y_{M-PF}(t)} < 1, \\
&a_{PF}(t+T) = \lambda_6 \cdot \min[1, \frac{\beta \cdot |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}] \cdot \dot{\theta}_M(t) + \lambda_7 \cdot \left\{1 - \min[1, \frac{\beta \cdot |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}]\right\} \cdot \dot{\theta}_{PL}(t) \\
&\quad - \lambda_8 \cdot \min[0, \theta_{DesPFF} - \theta_{PFF}(t)]
\end{align*}
$$

Where, $\dot{\theta}_M(t)$ and $\dot{\theta}_{PL}(t)$ are the visual angular velocity of Merging vehicle and PL, respectively, from the PF’s view at time $t$; $\theta_{PFF}(t)$ is the visual angle of PFF from the PF’s view at time $t$; $\theta_{DesPFF}$ is the desired visual angle from PF to its PFF, a constant value; $\lambda_4 - \lambda_8$ are parameters to determine the weights of stimuli in the models.

In equation 8, merging vehicle is the only leader of PF. The first term is for the car-following, and the second term is for the response of close PFF. In equation 9, PF follows both the PL and merging vehicle. The first term and the second term are for the car-following stimuli with relative longitudinal location resistance factor, and the third term is the response of close PFF behind PF.

4.2.3 acceleration-deceleration model of Putative Leader (PL)

The putative leader of merging vehicle follows its leader (PLL) in the merge process. When merging vehicle is close enough to PL, it would encourage the PL speeds up for cooperation to avoid possible collision in following lane changing. However, the probability of PL yielding action is affected by the relative longitudinal and lateral position of vehicles. In the lateral direction, the lateral separation between merging vehicle and PL could reduce the pressure coming from the merge process, which is contained in the visual angle information. In the longitudinal direction, the yielding possibility is lowered by the low relative longitudinal location...
factor $\gamma \frac{|X_{PL}(t) - X_{M}(t)|}{|X_{PL}(t) - X_{PF}(t)|}$, which reflects the longitudinal location adjustment ability of merging vehicle weighted by PL. $\gamma$ is a calibrated constant value. All the mentioned effects are expressed in equation 10:

$$a_{PL}(t+T) = \lambda_0 \cdot \theta_{PLL}'(t) - \lambda_{10} \cdot \min[0, \theta_{DesM} - \theta_{M}(t)] \cdot \min[1, \gamma \frac{|X_{PL}(t) - X_{M}(t)|}{|X_{PL}(t) - X_{PF}(t)|}]$$

where, $\theta_{PLL}'(t)$ is the visual angular velocity of PLL from the PL’s view; $\theta_{M}(t)$ is the visual angle of merging vehicle from the PL’s view at time $t$; $\theta_{DesM}$ is the desired visual angle from merging vehicle to PL, a constant value obtained from observation data. The first term expresses the PF follows its leader PLL and the second term represent the effects from the merging vehicle modified with the longitudinal effect factor.

5. MODEL TESTING AND RESULTS

5.1 Modeling Training and Testing Method
In the previous section, three dynamic acceleration-deceleration models of merging vehicle, PL and PF were proposed. In this section, these three models are calibrated and validated by U.S.101 observation data with a genetic algorithm in Matlab. The US101 data samples (398 valid merging processes) were randomly divided to two parts with equal numbers (199 samples each) for the model training and model testing separately. The time step in these models is set as 0.1 s according to the time step in NGSIM data. $T$ is the reaction time of drivers, and its value is increased from 0.5 s to 1.0 s with 0.1 s per running to search for the optimal results. The reaction time is usually set as 1.0 s in the conventional car following model, but in lane change model it may be shorter by considering that all the movements of vehicles are under drivers’ anticipation.

In the testing step, the observed stimuli at time $t$ are used as the input of models to get the predicted acceleration of testing samples at time $t+T$. The three acceleration and deceleration model are tested separately. When we conduct the training and testing of the merging vehicle acceleration-deceleration model, the observed characteristics of its PL and PF are used as the input of this model. The same process is applied for the testing of other two models.

Error test parameters used in this study are the mean error (ME), mean absolute error (MAE) and Theil’s inequality coefficient (U):

$$ME = \frac{1}{n} \sum_{i=1}^{n} Y_i^{sim} - Y_i^{obs}$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} |Y_i^{obs} - Y_i^{sim}|$$

$$U = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_i^{obs} - Y_i^{sim})^2}}{\sqrt{\frac{1}{n} \sum_{i=1}^{n} (Y_i^{obs})^2 + \frac{1}{n} \sum_{i=1}^{n} (Y_i^{sim})^2}}$$

Where, $Y_i^{obs}$ and $Y_i^{sim}$ are the observed and simulated acceleration of $i$th subject vehicle, respectively, with $i$ ranging from 1 to N.
5.2 Testing Results

The results of proposed acceleration-deceleration models are listed in Table 1 with different settings of driver’s reaction time.

<table>
<thead>
<tr>
<th>Reaction Time(s)</th>
<th>Errors (m/s²)</th>
<th>Merging Vehicle(M)</th>
<th>Putative Follower(PF)</th>
<th>Putative Leader(PL)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U-Train</td>
<td>ME-Train</td>
<td>MAE-Train</td>
<td>MAE-Test</td>
</tr>
<tr>
<td>0.5</td>
<td>0.82</td>
<td>1.14</td>
<td>1.61</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>0.72</td>
<td>0.40</td>
<td>1.01</td>
<td>1.05</td>
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<tr>
<td></td>
<td>0.81</td>
<td>0.56</td>
<td>1.18</td>
<td>1.16</td>
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<tr>
<td>0.6</td>
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<td>0.89</td>
<td>1.41</td>
<td>1.39</td>
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<tr>
<td></td>
<td>0.72</td>
<td>0.36</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>0.76</td>
<td>0.52</td>
<td>1.14</td>
<td>1.17</td>
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<tr>
<td>0.7</td>
<td>0.77</td>
<td>0.59</td>
<td>1.20</td>
<td>1.18</td>
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<tr>
<td></td>
<td>0.68</td>
<td>0.14</td>
<td>0.90</td>
<td>0.87</td>
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<tr>
<td></td>
<td>0.73</td>
<td>0.31</td>
<td>1.00</td>
<td>1.04</td>
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<td>0.8</td>
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<td>0.90</td>
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<tr>
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</tr>
<tr>
<td></td>
<td>0.82</td>
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<tr>
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<td>0.80</td>
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<td>0.36</td>
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<tr>
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<td>1.01</td>
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<tr>
<td></td>
<td>0.86</td>
<td>0.25</td>
<td>1.01</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*199 merging process samples for calibration of three models, 199 merging process samples for test of three models.

Based the simulation results shown in Table 1, reaction time $T$ should be set as 0.7 s for the sample set, which leads to smaller error comparing to other settings. The optimal reaction time is consistent with the result in reference 20 (0.667 s). The estimated results for the dynamic acceleration-deceleration models are expressed as follows:

For the merging vehicles:
\[ a_M(t + 0.7) = 2.415\theta_{PL}(t) - 4.570 \cdot \min[0, 0.065 - \theta_{PF}(t)] + 0.174 \cdot \min[0, 0.293 - \theta_L(t)] \]  
\hspace{1cm} (14)

For the putative followers (PF):

\[ a_{PF}(t + 0.7) = 4.460\theta_M(t) - 2.485 \min[0, 0.131 - \theta_{PF}(t)] \]  
\hspace{1cm} (15)

\[ a_{PF}(t + 0.7) = \frac{0.963 \cdot l}{\Delta Y_{M-PF}(t)} \begin{cases} 
1 - \min[1, \frac{0.789 |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}] \theta_M(t) \\
+2.873 \left[ 1 - \min[1, \frac{0.789 |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|}] \right] \theta_{PL}(t) \\
-2.661 \cdot \min[0, 0.131 - \theta_{PF}(t)] 
\end{cases} \]  
\hspace{1cm} (16)

For the putative leaders (PL):

\[ a_{PL}(t + 0.7) = 2.677\theta_{PLL}(t) - 5.972 \min[0, 0.074 - \theta_M(t)] \cdot \min[1, \frac{1.994 |X_{PL}(t) - X_{M}(t)|}{|X_{PL}(t) - X_{PF}(t)|}] \]  
\hspace{1cm} (17)

5.3 Discussion

The average absolute acceleration speeds of merging vehicle, PF and PL in our modeling time interval are 1.134 m/s^2, 0.840 m/s^2 and 0.947 m/s^2 in the sample set, respectively. Based on the error evaluation results shown in Table 1, we noticed the merging vehicles acceleration-deceleration model holds the highest error (MAE-Test = 1.18 m/s^2, U-Test = 0.77) comparing to other two models. The possible explanation is that merging vehicles bear the most complex workloads, maintaining route, keeping safe distance between surrounding vehicles, preparing for lane changing, which increases the difficulty to predict their reaction. The PF acceleration-deceleration model has the best performance with the lowest error value (MAE-Test = 0.87 m/s^2, U-Test = 0.68). It may result from that the PF could rationally react to the effects of other vehicles with more accurate information obtained by looking ahead, which makes the simulation more realistic by rational equations in physics. Overall, the omitting of heterogeneity of drivers may contribute to the errors of the proposed models. Adding the attributes of drivers into the model frame will be considered in our future study.

For the merging vehicle acceleration-deceleration model, we could tell that the merging vehicles are more sensitive to the distance between itself and its PF comparing to the one between itself and its L, as \( \lambda_2 \) (4.570) is much larger than \( \lambda_3 \) (0.174). The calibrated desired visual angle for PF (0.065) is smaller than that for L (0.293), which means the merging vehicles need larger space between its putative leader and itself, and they could endure smaller space gap between its leading vehicle on current lane and itself. The conclusion from the calibrated model is reasonable. Because that the merging vehicles is particularly sensitive to their relative location to their PF for the anticipated merge maneuver and driving on main lane. However, the safety issue coming from the leader in current lane is an intermediate constraint, which will be terminated with successful merge maneuver.

For the PF acceleration-deceleration model, based the calibration results, when the lateral distance between the merging vehicle and PF is larger than 3.23 m (0.963 times the length of the
lane width), the PF would follow both merging vehicle and PL. Otherwise, the merging vehicle 
would not sense the effect of the PL. It makes sense that when the merging vehicle become 
closer to the mainline (considering the shift away of PF), the PF could feel the strong merge 
intention of it and set it as potential lead. The desired visual angle from PF to its follower (PFF) 
is 0.131, which is higher than the required value from merging vehicles to PF (0.065). This 
means the required space distance from PF to its follower is smaller, which accord with the real 
condition that lane changing vehicles need more space for safety consideration. The calibration 
result of $\beta$ is a small value (0.789) as we expected, which indicates the PF holds resistance on 
dramatic deceleration as it approaches the merging vehicle due to $\frac{0.798 \cdot |X_M(t) - X_{PF}(t)|}{|X_{PL}(t) - X_{PF}(t)|} \ll 1$.

For the PL acceleration-deceleration model, the PL has intensive reaction to the merging 
vehicle with a high coefficient (5.972) and low desired visual angle (0.074), which indicates the 
influence of merging on the PL exists. The calibration result of $\gamma$ is 1.994, which means the 
acceleration resistance of PL to the nearby merge vehicle would disappear when the merging 
vehicle almost locate in the middle of PL and PF along the longitudinal direction. To evaluate 
the effect of merging vehicle on the PL, a basic acceleration-deceleration model of PL is built 
without stimuli coming from the merging vehicle, written as:

$$a_{PL}(t + T) = \lambda_o \cdot \theta_{PLL}(t)$$

(18)

After training and testing, the following error test results were obtained for the base 
model: MAE-Train = 1.27 m/s², MAE-Test = 1.28 m/s² and U = 0.78. Referring to Table 1, it 
could be concluded that introducing the impact of merging vehicle to the PL acceleration-
deceleration could improve the simulation accuracy by 19.5%. The testing and comparison 
results are in accordance with the observation results in section three.

Figure 5 shows the simulated acceleration of sample vehicles, which were also showed in 
Figure 2 as examples. From the comparison of the simulated results and the observed results, it 
tells that the proposed model has the capability to reflect the interactions among vehicles in the 
merge process. When the merging vehicle approach its PL or PF, the PL/PF responds with 
yielding action whenever it is needed in the congested area. However, some further 
improvements are required in our models for higher accuracy by introducing the characteristic 
and random selection behavior of different drivers.
6. CONCLUSION

Although interactions between drivers are essential for modeling lane changes, they have not been incorporated explicitly into existing microscopic traffic flow model. The most significant contribution of the present study is the introduction of the yield behavior of PL and the lateral separation between vehicles (both features were largely ignored in the past) into lane change models to help capture the interactions between vehicles.

The study findings indicate some considerable conclusions in lane change process. First, obvious yield behavior of PL and PF exist in congested merge area, while in the past only the yield behavior of PF has been incorporated into the lane change modeling. Thus, the stimuli coming from the front and behind of the subject vehicles were both implemented in the proposed acceleration-deceleration models. Second, lateral separation between vehicles can not be ignored during the lane change process especially in a merge section. The proposed acceleration-deceleration models used visual angle information to extend stimuli into two-dimensional space, which can practically explain complex driving behavior with lateral movement. Third, merging vehicle, PL and PF all could clearly see the relative locations of themselves and the merging vehicle could do plenty of longitudinal location adjustment before lane change maneuver, which is significantly different from the conventional car-following models (vehicles drive in the center of one lane) and influence the yielding decision of PL and PF. To capture the particular car-
following behavior in lane change process, the relative longitudinal location factors are introduced in our models.

Finally, the model calibration and validation results based on the U.S.101 data demonstrates that the proposed acceleration-deceleration models could qualitatively predict the driving behavior of vehicles in the lane change process and obtain acceptable training and testing errors. To illustrate the advantages of incorporating the yield behavior of PL into models, the proposed PL model was compared with a basic PL acceleration-deceleration model only under the influence of PLL, and the results shows introducing the influence of the merging vehicle on its PL could more mimic the driving behavior of PL.

Nonetheless, the heterogeneity of the drivers is not incorporated in the present study, which is due to the lack of drivers’ information in the NGSIM data. Further experimental data collection is worth executed to improve the models and increase prediction accuracy in the future. This paper only focuses on modeling the driving behavior of vehicles for the accepted gap. In future, we will try to build a more comprehensive lane change model including the rejected gap by parameterizing the gap selection.

REFERENCE


