Bi-directional Control Characteristics of General Motors (GM) and Optimal Velocity Car-Following Models: Implications for Coordinated Driving in Connected Vehicle Environment

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ABSTRACT
In natural traffic flow, the information from preceding vehicles determines driver behavior predominantly. With the availability of connected vehicle technologies (CVT), drivers can receive information from both preceding and following vehicles. This creates new opportunities for vehicle coordination and control at the microscopic level based on bi-directional information. Although the bi-directional car following models have been studied since the 1960s, most existing car-following models, especially those used by adaptive cruise control (ACC) technologies, are still forward-only car-following models. This paper serves as a first step towards the use of bi-directional car-following models for microscopic vehicle coordination and control. The focus is on studying their general control characteristics and their impact on traffic flow stability. A general bi-directional control framework is proposed to convert any car-following model into bi-directional forms. Four representative GM (General Motors) and optimal velocity car-following models are reformulated and calibrated against field vehicle trajectory data collected in the NGSIM (Next Generation SIMulation) project. The bi-directional control characteristics of the selected models are evaluated by tuning the percentage of the consideration of backward information in the final car-following decision. The evaluation uses forward versus backward acceleration diagrams and a ring road stability analysis with respect to equilibrium states obtained from the NGSIM data. The increase of backward information contribution may help alleviate traffic congestion and stabilize traffic flow. Meanwhile, an operating range of backward information contribution between 5-20% is recommended so that the resulting model can produce reasonable results for both free flow and congestion situation.

KEYWORDS: CAR-FOLLOWING MODELS, FORWARD AND BACKWARD CONTROL, BI-DIRECTIONAL CONTROL, NGSIM TRAJECTORY DATA
NOTATIONS

1. \( x_n(t) \): the location of the front bumper of vehicle \( n \) at time \( t \).
2. \( v_n(t) \): the velocity of vehicle \( n \) at time \( t \).
3. \( a_n(t) \): the acceleration of vehicle \( n \) at time \( t \).
4. \( x_{n-1}(t) \): the location of the front bumper of the preceding vehicle \((n-1)\) of vehicle \( n \).
5. \( v_{n-1}(t) \): the velocity of the preceding vehicle \((n-1)\) of vehicle \( n \) at time \( t \).
6. \( a_{n-1}(t) \): the acceleration of the preceding vehicle \((n-1)\) of vehicle \( n \) at time \( t \).
7. \( x_{n+1}(t) \): the location of the front bumper of the following vehicle \((n+1)\) of vehicle \( n \) at time \( t \).
8. \( v_{n+1}(t) \): the velocity of the following vehicle \((n+1)\) of vehicle \( n \) at time \( t \).
9. \( a_{n+1}(t) \): the acceleration of the following vehicle \((n+1)\) of vehicle \( n \) at time \( t \).
10. \( s_n(t) \): \( x_{n+1}(t) - x_n(t) \), the spacing between vehicle \((n-1)\) and vehicle \( n \) at time \( t \).
11. \( \Delta v_n(t) \): \( v_{n+1}(t) - v_n(t) \), the velocity difference between vehicle \((n-1)\) and vehicle \( n \) at time \( t \).
12. \( \lambda, \lambda_1, \lambda_2 \): the sensitivity coefficients.
13. \( \tau \): the reaction time.

INTRODUCTION

This paper explores the bi-directional control characteristics of car-following models that may be used for microscopic vehicle coordination and control in future driving environment with connected vehicles.

Connected Vehicle Technology (CVT) is an emerging ITS technology that has the potential to reshape the prevailing transportation systems. CVT uses wireless communication technologies to enhance the V2V (Vehicle to Vehicle), V2I (Vehicle to Infrastructure), and I2I (Infrastructure to Infrastructure) connectivity resulting in considerable potentials for a variety of applications to promote safety, mobility, and sustainability. In this environment, drivers not only receive stimulus from the directly preceding vehicles but also from vehicles that are within CVT communication range; while, in prevailing driving environment, drivers only perceive the preceding vehicle states constantly and occasionally check the following vehicles through side and rear mirrors. Further into the future, with the enhanced connectivity, vehicle-specific traffic control or guidance based on the surrounding traffic conditions may become possible. However, to establish theoretical and modeling foundations for such future technologies, it is necessary to explore the characteristics of traffic flow under the CVT environment. Such characteristics can be described from two aspects, the increased perception range and the bi-directional information propagation. Furthermore, we consider that the latter one has a more profound impact on the characteristics of traffic flow than the former because traffic flow has long been considered a hyperbolic and anisotropic flow in which information propagate from upstream to downstream at a finite speed. Microscopic vehicle control and coordination technologies have been studied for many years. A representative this technology is the Adaptive Cruise Control (ACC) technology. The prevailing trend of ACC is the integration of microscopic traffic flow models into the control framework of ACC \((l)\). However, one limitation in existing models is that the underlying car-following models only considers information from the preceding vehicles. Recently, Yang and Recker \((2)\) explores the ACC technology under CVT environment; however, macroscopic traffic information, such as congestion and incidents, was only considered, rather than the microscopic dynamics of surrounding vehicles. In this study, we take the first step towards the possibility of using bi-directional car-following models in microscopic vehicle coordination and control technologies such as ACC by evaluating the control characteristics of several representative bi-directional car-following models.

Furthermore, CVT is still at an early stage and will need many years of research, innovation, and marketing to achieve successful large-scale implementation as well as full penetration among all vehicles. During this process, traffic streams will likely consist of both regular and connected vehicles. In our study, we intend to explore models that may be used in these hybrid environments without introducing severe driver confusion and vehicle conflicts. While such models may exhibit vehicle characteristics similar to regular vehicles, they can also be tuned and modified to apply effective vehicle control or coordination.
strategies. Among the existing microscopic traffic flow models, bi-directional microscopic models can serve both purposes well. They can be calibrated to reflect the driver behavior of regular vehicles, while also being tuned to apply microscopic control and coordination strategies rooted in the bi-directional information propagation through the connected vehicles. In the existing literature, traffic flow models that can describe such bi-directional impacts can be found in both the car-following models and cellular automation models. In car-following modeling, bi-directional extensions to the original unidirectional, one-leader-one-follower models can be found in literature as early as the 1950s. Herman et al. (3) first proposed a forward and backward control car following formulation to investigate the stability of several variations to the GM (General Motors) car following models. Nakayama et al. (4) and Hasebe et al. (5) extended the optimal velocity (OV) models to bi-directional forms. More recently, Ge et al. (6) and Sun et al. (7) further extend the bi-directional formulations of OV models to any arbitrary number of preceding and following vehicles to simulate the impact of ITS technologies on vehicle dynamics. Both studies conducted analytical and numerical linear stability analysis; however, neither utilizes field data to validate their models.

In this paper, a generalized bi-directional formulation is presented. Four representative car-following models, two GM and two OV models, are reformulated using the proposed framework. The proposed framework can be easily applied to other more complicated car-following models such as CA (Crash Avoidance) and AP (Action Point) models as long as the backward control counterparts can be developed (8). The relationship between the new bi-directional formulations and the existing formulations are also discussed in detail. All four models are calibrated and evaluated using field vehicle trajectory data obtained from the NGSIM (Next Generation Simulation) project. The forward versus backward control variable diagram and the numerical ring road stability analysis are used to evaluate the bi-directional control characteristics of each model. A Sensitivity analysis is also conducted by varying the contribution of backward information.

**LITERATURE REVIEW**

In this section, the existing studies related to the proposed bi-directional formulations are reviewed. First, four unidirectional car-following models are reviewed including the original relative velocity only model (9) and its full version (10), the original OV model (11), and its Full Velocity variation (12).

**Selected GM and OV Car-Following Models**

GM models assume that the follower vehicle \( n \) adjusts its acceleration based on traffic state variables within the leader-and-follower vehicle system, including the relative velocity, vehicle spacing, and vehicle velocities. The first GM model proposed by Chandler, Herman and Montroll (9) only considers relative velocity.

\[
a_n(t + \tau) = \lambda \ast [v_{n-1}(t) - v_n(t)]
\]  \hspace{1cm} (1)

The complete version of GM model is proposed by Gazis et al. (10).

\[
a_n(t + \tau) = \frac{\lambda v_n(t)^m [v_{n-1}(t) - v_n(t)]}{s_n(t)^l}
\]  \hspace{1cm} (2)

where \( l \) and \( m \) are both exponents applied to the following vehicle velocity and spacing, respectively.

The first OV model is proposed by Bando et al. (11). The OV model assumes each vehicle pursues an optimal velocity that is dependent on the vehicle spacing with the preceding vehicle. The formulation is as followings:

\[
a_n(t + \tau) = \lambda \ast [V(s_n(t)) - v_n(t)]
\]  \hspace{1cm} (3)

where \( V(s_n(t)) \) denotes the OV function. Bando et al. (11) also recommended a formulation for \( OV(-) \) which converts Equation 2 into the following:

\[
a_n(t + \tau) = \lambda \ast \left\{ V_0 \ast \left\{ \tanh \mu(s_n(t) - b_f) - \tanh \mu(b_c - b_f) \right\} - v_n(t) \right\}
\]  \hspace{1cm} (4)
Jiang (12) improved Bando’s model by adding the velocity difference between vehicles \( n \) and vehicle \((n-1)\).

\[
a_n(t + \tau) = \lambda_1 \left[ V_1 + V_2 \tanh \left[ c_1 (s_n(t) - L_{n-1}) - c_2 \right] - v_n(t) \right] + \lambda_2 (v_{n-1} - v_n)
\]

(5)

**Existing Bi-Direction Control Formulations**

In 1959, Herman et al. (3) introduced the “backward control” formulation to the GM Models. The formulation is given by the following:

\[
a_n(t + \tau) = C_1 [v_{n+1}(t) - v_n(t)] + C_2 [v_{n+1}(t) - v_n(t)]
\]

(6)

The reaction time here has been assumed to be the same for both forward and backward control. Herman further provided the analytic solution to this equation and analyzed its stability conditions in a three-vehicle system. The major advantage of this model is that the forward and backward terms are clearly separated from one another. The limitation with this formulation is that the values of \( C_1 \) and \( C_2 \) are affected by both the sensitivity for each direction and their contributions in the final car-following decision. This stability condition is as follows:

\[
\frac{(C_1 - C_2)}{(C_1 + C_2)} < \frac{1}{2}
\]

(7)

Another existing bi-directional formulation is the backward-looking optimal velocity (BL-OV) model (4).

\[
a_n(t + \tau) = \lambda [V_{s}(s_n(t)) + V_{s}(s_{n+1}(t)) - v_n(t)]
\]

(8)

where \( V_{s}(x) \) are the OV functions for forward-looking and backward-looking, respectively. Both terms together are used to replace the \( V(x) \) term in the original OV model. Furthermore, the formulation of the \( V_{s}(x) \) and \( V_{b}(x) \) take the following forms:

\[
\begin{align*}
V_{s}(x) &= \beta \left[ \tanh(x - \gamma) + \chi \right] \\
V_{b}(x) &= -\beta \left[ \tanh(x - \gamma) + \chi \right]
\end{align*}
\]

(9)

The stability condition for this bi-directional formulation is also given.

\[
\lambda > 2 \left[ \frac{V_{s}(b) + V_{b}(b)}{V_{s}(b) - V_{b}(b)} \right] \frac{2(\beta_{s} - \beta_{b})}{\cosh \left( b - \gamma \right) (\beta_{s} + \beta_{b})}
\]

(10)

where \( b \) is the mean spacing that may occur in the traffic flow. Hasebe et al. (5) later extended the model to an arbitrary number of preceding and following vehicles.

Ge et al. (6) also proposed a bi-directional OV model for multiple preceding and a single following vehicle. However, in Ge et al. (6)’s model, the OV function was simplified into a linear combination of individual OV functions with respect to each vehicle considered. A percentage factor, \( \alpha \), was used to quantify the impact of the forward and backward decisions. If only one preceding and following vehicle is considered, their model can be written as the following:

\[
a_n(t + \tau) = \lambda \left[ \alpha V_{s}(s_n(t)) + (1-\alpha) H(h_{n} - s_{n+1}(t)) V_{b}(s_{n+1}(t)) - v_n(t) \right]
\]

(11)

where \( \alpha \) is used to identify relative roles of the two OV functions in car-following decisions, \( H(x) \) is a Heaviside function in which \( H(x) = 1 \), if \( x > 0 \), and \( H(x) = 0 \), otherwise, and \( h_{n} \) is an interaction distance within which drivers respond to stimulus from following vehicles. The stability condition is given assuming the same formulation for the forward and backward OV function in Nakayama et al.’s model.

\[
\lambda > 2 \left[ \frac{\alpha V_{s}(b) + (1-\alpha) V_{b}(b)}{\alpha V_{s}(b) - (1-\alpha) V_{b}(b)} \right] \frac{2(\alpha \beta_{s} - (1-\alpha) \beta_{b})}{\cosh \left( b - \gamma \right) (\alpha \beta_{s} + (1-\alpha) \beta_{b})}
\]

(12)

Sun et al. (7) extended the full velocity OV model into a similar form that considers multiple preceding and one following vehicle. Their model can be rewritten as follows when only one preceding vehicle and one following vehicle is considered.

\[
a_n(t + \tau) = \alpha (\lambda V_{s}(s_n(t)) + \kappa (v_{n+1}(t) - v_n(t))) + (1-\alpha) H(h_{n} - s_{n+1}(t)) (\lambda V_{b}(s_{n+1}(t)) + \kappa (v_n - v_{n+1})) - \lambda \dot{v}_{n}(t)
\]

(13)

Their multi-vehicle stability condition can be simplified into the following:
\[
\frac{1}{\lambda} < \frac{\alpha v^*_r(b) - (1-\alpha)v^*_f(b)}{2[\alpha v^*_r(b) + (1-\alpha)v^*_f(b)]} + \frac{2\kappa}{\alpha v^*_r(b) + (1-\alpha)v^*_f(b)}
\]

(14)

It can be observed that when \( \kappa = 0 \), Equation (14) becomes (12).

**METHODOLOGY**

**The Generalized Bi-Directional Framework**

Each existing bi-directional model has its own benefits and limitations to evaluate the bi-directional information impact on traffic flow. Combining those benefits together, we propose a generalized bi-directional framework that can be used to formulate all car-following models into their bidirectional forms. Two different types of variables are considered to formulate the generalized bi-directional framework, the perception variables \( x \) and the decision variable \( u \). Furthermore, we consider that the bi-directional car-following control task can be divided into the forward-looking and backward-looking subtask. Each subtask has its own perception-reaction characteristics that are independent of one another. Decisions from both directions are then combined to produce a driver’s final car-following decision with the vector of perception variables denoted as \( x \) and the decision variable as \( u \). A car-following model can be generalized as \( u = f(x) \). The general bi-directional framework can be written as follows:

\[
u = \alpha \cdot f_F (x_F) + (1 - \alpha) \cdot f_B (x_B)
\]

(15)

That is,

\[
u = \alpha \cdot f_F (x_F) + (1 - \alpha) \cdot f_B (x_B)
\]

(16)

where \( F \) and \( B \) indicate the forward- and backward-looking terms respectively, and \( \alpha \) indicates the percentage of forward control contribution in a driver’s final decision with \( 0 \leq \alpha \leq 1 \). The framework incorporates two useful characteristics from the existing bi-directional models including the separated and independent forward and backward control submodels from Herman et al.’s model, and the linear combination between forward and backward control (6, 7). The Heaviside function used in Ge et al. (6) and Sun et al. (6) is considered as one part of the backward control model. The proposed framework can be used to reformulate the four selected car-following models into the bi-directional forms (See Table 1).

**TABLE 1 Formulations of Seven Selected Car-Following Models for bi-directional control**

<table>
<thead>
<tr>
<th>Model</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandler</td>
<td>( a_i(t+\tau) = \alpha \lambda [v_{x,i}(t) - v_i(t)] + (1-\alpha) \lambda [v_{x,i}(t) - v_i(t)] )</td>
</tr>
<tr>
<td>GM</td>
<td>( a_i(t+\tau) = \alpha \frac{\lambda v^<em><em>r(t)}{s_i(t)}[v</em>{x,i}(t) - v_i(t)] + (1-\alpha) \frac{\lambda v^</em><em>f(t)}{s_i(t)}[v</em>{x,i}(t) - v_i(t)] )</td>
</tr>
<tr>
<td>Bando</td>
<td>( r^<em>_i(t+\tau) = \alpha \lambda [v^</em>_r(t)] + (1-\alpha) r^*_i(t) ) |</td>
</tr>
<tr>
<td>Jiang</td>
<td>( a_i(t+\tau) = \alpha \lambda [v^<em>_r(t)] + (1-\alpha) r^</em>_i(t) ) |</td>
</tr>
</tbody>
</table>

**Car-following Model Adjustment and Calibration**

Classic car-following models, like the GM and OV models, do not have the built-in mechanism to suppress unrealistic events such as crashes, excessive speed or acceleration, and backward movements. For those models to be used in real-world vehicle coordination and control systems, the models need to reflect real-world car-following dynamics. In this study, we developed an algorithm to reduce crash rates and prevent the backward movements. At each time step, the algorithm is executed in turn from the second vehicle to the last in a car-following platoon designed to mimic the execution of car-following in a
micro-simulation software. The intent of the algorithm is to develop the upper and lower bound of the vehicle velocity so that the following criteria are satisfied:

a. The following vehicle does not crash into the preceding vehicle at the next time interval.
b. Vehicle acceleration does not exceed maximal acceleration $a_{\text{max}}$.
c. Vehicle deceleration does not exceed maximal deceleration $b_{\text{max}}$.
d. Vehicle velocity does not exceed a reasonable maximal speed (e.g. 20 mph above speed limit).
e. Vehicle velocity does not go backwards.

Since contradictions from the above criteria may result in a lower bound that is greater than the upper bound for velocity, the contradictions are to be executed sequentially following the above order. Thus the more important criteria, such as preventing backward movements and ensuring velocity and acceleration bounds, are enforced more stringently than the other criteria, e.g. crash conditions, which may occur in reality. The convergence in the calibration is critical for this study due to relatively large number of parameters in bi-directional car-following models. Car-following model calibration is a typical numerical optimization problem. The objective function of the problem is a function of the estimation error with respect to ground truth data. The variables to optimize are the car-following model parameters. The constraints are the boundaries of model parameters. Given the complicated structure of the solution surface of model parameters, car-following models are usually calibrated by using intelligent algorithms. In such algorithms, the objective function is critical to ensure the optimality and convergence. An earlier study conducted by the authors (13) indicates the following objective function: weighted velocity mean absolute error (WvMAE) in combination with the Genetic Algorithm (GA) is the most effective way of obtaining model parameters that do not cause significant error accumulation in multistep car-following simulations.

$$WvMAE = \frac{1}{N} \sum_{i=1}^{N} \left( \phi^p | \hat{v}_i - v_i | \right)$$

where $w_i = \phi^p$ is the weight applied to the $i$th simulation result, $p_i$ is the number of car-following calculations made, and $\phi$ is the base of the exponential function to be calibrated together with the model parameters. The calibrated models are further inspected using the multistep acceleration, velocity, and location error curves.

This study uses the detail vehicle trajectory data collected at the southbound direction of U. S. Highway 101 (Hollywood Freeway) in Los Angeles, California from the NGSIM project (14). The entire segment is approximately 2100 feet in length, with five main lanes throughout the section and one auxiliary lane. The entire data collection time period is 45 minutes between 7:50 to 8:35 a.m. during the morning peak hours. Vehicle trajectory data has a temporal resolution of 0.1 second. A Java program extracts 1980 groups of 30-second trajectories for three consecutive following vehicles. In our study, the reaction time is used for the time interval for running the car-following models as is done in many existing studies (15). The reaction time of 1.1 seconds is selected from the normal range of reaction times between 0.88 to 1.51 seconds (16). The extracted dataset is divided into the training and testing datasets. The performance of the calibration results are inspected using the average MAEs over multiple time steps. It should be noted that the proposed bi-directional model will ultimately be used in the driving assistance systems such as speed or spacing warning or ACC in each individual vehicle. In this study, we calibrate only one set of model parameters for all vehicles traveling on the US101 section rather than a different set of model parameters for each individual vehicle since we are more interested in a generally reasonable range of model parameters rather than some extreme parameter values of some individual drivers. Furthermore, the number of car-following trajectory points that can be obtained through this short US101 section may not be sufficient to effectively calibrate all the model parameters in the bi-directional models.
**Bi-Directional Control Characteristics Evaluation**

*Forward versus Backward Diagram*

In the general bi-directional formulation (Equation 6 and 7), $\alpha$, $\beta$ are formulated as weights reflecting a driver’s consideration on traffic information from both upstream and downstream respectively. However, after calibration, it is difficult to distinguish their values from the sensitivity coefficients of each car-following model. In fact, these two weights serve more as a control vector rather than a measuring vector of the bi-directional contributions. Nevertheless, the bi-directional characteristics can still be analyzed by evaluating the following equation.

\[ u = u'_F + u'_B \]  

where $u'_F = \alpha \cdot u_F$ and $u'_B = \beta \cdot u_B$ are the combined values of the forward- and backward- decision values multiplied by their weights. Further, each sample of a car-following vehicle group in a single time step can generate one $(u'_F, u'_B)$ observation based on the car-following models calibrated. When plotting this data on the $u'_F$ versus $u'_B$ diagram, the control characteristics of each car-following model can be analyzed based on the distribution of $(u'_F, u'_B)$ points. In this study, all models have been converted into the acceleration form; thus, acceleration is the state variable used in this analysis.

**Pseudo Ring-Road Linear Stability (LS) Test**

The stability condition of the new bi-directional formulations can be easily derived based on the conditions for the existing formulations by evaluating the relationship between the parameters in the new formulation and those in the existing ones. While it is difficult to incorporate into the analytic linear stability analysis for the additional models introduced in previous sections to suppress crash rates and backward moving, the numerical experiments of the linear stability analysis, the ring-road experiments, can still be applied. Meanwhile, NGSIM dataset provides real-world vehicle trajectories. However, lane changing models need to be formulated to conduct the multilane simulation. Combining two models can create confusion in the evaluation results since lane changes can also be significantly affected by bi-directional information propagation. In this study, to take advantage of both the numerical linear stability analysis and the NGSIM dataset, a pseudo ring-road linear stability (LS) test is proposed. This test is conducted on a pseudo ring road with its perimeter being the same as the length of the vehicle platoon to be investigated. Assuming for a $p$-vehicle platoon, at any given instance $t$ the perimeter of the PRSS ring road $C_{prss}(t)$ can be calculated as the following.

\[ C_{prss}(T) = x_1(T) - x_p(T) \]  

where $x_1(T)$ and $x_p(T)$ are the locations of the first and the last vehicle in the $p$-vehicle platoon, respectively. At the initial equilibrium state, all vehicles have the same velocity and spacing as the average velocity and spacing in the field data along with zero acceleration. The “ring” is completed by merging the first and the last vehicle at their front bumper locations. In this way, the initial equilibrium state on ring road is established based on real conditions. A perturbation of spacing reduction of half the average spacing and a velocity increase of 15 ft/s (about 10 mile/h, 4.57 m/s) is applied to the middle vehicle in the platoon. Then similar to the linear stability analysis, the car-following models are executed iteratively for a large number (e.g. 500) of time intervals with respect to the initial condition of the $p$-vehicle system at time $t$. The resulting trajectory plots are evaluated for the flow stability.
RESULT ANALYSIS

Calibration Results

Figure 1 illustrates the accumulative error results of the calibrated models. Each data point \((n, MAE_{a,v,x})\) in the diagram is the acceleration, velocity, or location MAE for all simulation results that require \(n\)th car-following calculations to be obtained. The trend of these curves illustrates the convergence of simulation errors across multistep car-following calculations. As indicated in the three diagrams in Figure 1, the acceleration, velocity, and location MAE for each algorithm converges indicating a goodness of fit for the calibrated model with the actual car-following behavior. The calibrated parameters are listed in Table 4. It should be noted that the calibrated \(\alpha\) ranges from 80% to 93% indicating significant dominance of forward information in real-world traffic with minor backward information perceived likely through the use of back and side mirrors.
## TABLE 2 Calibrated Bi-Directional Model Parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>Forward Control Parameters</th>
<th>Backward Control Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>( \lambda )</td>
</tr>
<tr>
<td>Chandler</td>
<td>0.800</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( \lambda )</td>
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</tr>
<tr>
<td>GM</td>
<td>( \alpha )</td>
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</tr>
<tr>
<td></td>
<td>( \lambda )</td>
<td>0.707</td>
</tr>
<tr>
<td></td>
<td>( m )</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>( l )</td>
<td>-0.062</td>
</tr>
<tr>
<td>Bando</td>
<td>( \alpha )</td>
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</tr>
<tr>
<td></td>
<td>( \lambda )</td>
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</tr>
<tr>
<td></td>
<td>( V_0 )</td>
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<tr>
<td></td>
<td>( \mu )</td>
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<td>( b_c )</td>
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<tr>
<td>Jiang</td>
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</tr>
<tr>
<td></td>
<td>( \lambda_1 )</td>
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<tr>
<td></td>
<td>( \lambda_2 )</td>
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<td>( V_1 )</td>
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<td></td>
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<tr>
<td></td>
<td>( c_2 )</td>
<td>4.003</td>
</tr>
</tbody>
</table>

### Forward versus Backward Acceleration Characteristics

Figures 2 and 3 illustrate the forward versus backward acceleration diagrams for different \( \alpha \) values. In each diagram, the color coding indicates the density of \((a_f, a_B)\) within each 0.1 ft/s (0.07 m/s) by 0.1 ft/s (0.07 m/s) grid. Figure 2a illustrates the actual conditions in which the distributions follow an eclipse shape with longer axis at the vertical position. The distribution of Bando and Jiang’s algorithm shifts towards the right indicating that the backward acceleration provides more acceleration decisions than deceleration decisions. This is a reasonable pattern because drivers are more concerned about the pressure from fast-moving following vehicles than the relaxation posed by slow-moving following vehicles. The \((a_f, a_B)\) with high backward information contribution (e.g. 30%, 50%, and 90%) distributed irregularly in the diagrams indicate that the parameters calibrated for the low backward information may have issues when operating at high backward information control scenarios. Chandler’s model maintains the best distribution shape among all the algorithms. The GM model falls into reasonable shape when the backward information contribution is below 30%. Bando and Jiang’s model both exhibits irregular distribution above 20% backward information levels.

a. Actual Condition  

b. 90% Forward Information  

c. 50% Forward Information  

d. 30% Forward Information  

FIGURE 2 Density-contour plot of forward versus backward acceleration (actual, 30-90% backward information).
Pseudo Ring-Road LS Test Results

Figures 4 and 5 further demonstrate the stability characteristics with respect to different backward information contribution levels. The individual points in each diagram are the trajectory points for the vehicles in the pseudo ring-road. When congestion builds up, vehicle spacing becomes smaller, and the trajectory points form clusters in the diagrams. During free flow condition, the vehicle spacing is large and more scattered areas occur. The trajectory diagram has been widely used in linear stability analysis to examine congestion propagation. In this test, we establish two initial equilibrium conditions based on the entire vehicle sequence on the leftmost general purpose lane of the US101 NGSIM data at 8:05 am and 8:29 am. The former is in a free-flow condition, while the latter is in a congested condition. At 8:05 am, vehicles on the leftmost lane were running at an average velocity of 43.0 ft/s (29.3 mile/h, 47.2 km/h) and an average spacing of 88.2 ft (26.9 m). The average vehicle velocity at 8:29 am was 18.3 ft/s (12.5 mile/h, 20 km/h) and the average spacing was 59 ft (18.0 m). The size of the congested area (CA) is also reported. This area size is calculated by the total size of 10 sec by 10 ft (3.04 m) grids that have an average vehicle velocity below 30 mph (48.3 km/h).
Figure 4 demonstrates that significant traffic waves may occur in the LS test when the initial perturbation is added to the middle vehicle even in a free-flow vehicle platoon. Traffic waves occur in most scenarios with the exception of Jiang’s model under the low backward information contribution and the GM model under 5% backward contribution. Based on the CA value, the best overall traffic condition is achieved at 30% and 10% backward contribution for the Chandler model, 0% for the GM model, 0% for the Bando model, and below 5% for Jiang’s model. This indicates that in general during free-flow condition, the consideration of information from backward may not be necessary. Furthermore, the GM
model shows relatively better traffic condition when the backward contribution is below 5% and greater
than 30% despite traffic condition for 5% to 30% becomes quite congested. This may raise a concern for
using the GM model for microscopic vehicle coordination.

**FIGURE 5** Pseudo ring-road LS test results for congested condition.

Figure 5 shows the LS tests results for a congested initial state. Despite the initial congestion, the
traffic condition on the pseudo ring road after 500 seconds may still achieve high average velocity
although accompanied by traffic waves. The initial perturbation further adds additional instability to the
already congested ring road. The resulting congestion cannot be withstood by GM and Jiang’s models at
high backward contribution rate and most of the vehicles in the platoon stop. It should be noted that
although the GM model produces smoother traffic flow than the other three models, most vehicles are traveling at low speed resulting in congestion over the entire space-time diagram ($CA = 358.44\text{ km*sec}$). This phenomenon indicates the backward information contribution may need to limited to a reasonable range to avoid failures in microscopic coordination and control. The best congestion wave result is achieved for 10% backward contribution rate for Chandler’s model and Jiang’s model; while the best result is obtained at 5% for Bando’s model. Since the backward control component in Jiang’s model does not produce any positive impact in congestion reduction, the model may not be suitable to the state propagation model in microscopic coordination and control.

CONCLUSION AND FUTURE WORK
In this study, we explore the bi-direction control characteristics of four representative GM and optimal velocity car-following models to be used for microscopic vehicle coordination and control in the Connected Vehicle environment. A general bi-directional framework is proposed to reformulate any car-following models into the bi-directional forms. The proposed framework is used to convert Chandler, GM, Bando, and Jiang’s models into their bi-directional forms and is calibrated using US101 NGSIM vehicle trajectory data. The forward versus backward acceleration diagram and a pseudo ring road linear stability test are used to evaluate traffic flow characteristics under different backward information contribution level.

The forward versus backward acceleration curve show that car-following parameters calibrated at a low backward information contribution level may not be suitable for operating at high backward information contribution levels. The pseudo ring road LS test for the free flow condition indicates that incorporating backward information may not necessarily improve traffic conditions in light traffic flow. The LS test for congested condition indicates that backward information contribution can help reduce traffic congestion and traffic waves. In general, Chandler’s and Bando’s models provide better operating performances when compared to the other two models. A recommended operating range between 5-20% is suggested for tuning the backward contribution for vehicle coordination and control.

Future work of this study includes several aspects. First, representative car following models in other categories such as crash avoidance and action point models need to reformulated and investigated. The initial selection of models is based on whether the model’s bidirectional stability has been studied in the existing literatures. Nevertheless, the bi-directional formulation framework and the evaluation methods proposed in this paper can be applied to other car following models even when the analytic linear stability condition is difficult to obtain. Second, the selected bi-directional car following models need to be integrated into existing Adaptive Cruise Control (ACC) or other microscopic vehicle coordination and control framework to evaluate their effectiveness more meticulously.

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